Dividing Polynomials

The **Quotient of Powers** rule states that for any nonzero number a and all integers m and n, $\frac{a^m}{a^n} = a^{m-n}$.

To find the power of a quotient, find the power of the numerator and the power of the denominator.

EXAMPLE

1 **Simplify.**

a. $\frac{x^5y^8}{-xy^3}$

$$\frac{x^5y^8}{-xy^3} = \left(\frac{x^5}{-x}\right)\left(\frac{y^8}{y^3}\right)$$ Group powers that have the same base.

$$= -(x^{5-1})(y^{8-3})$$ Quotient of powers

$$= -x^4y^5$$ Simplify.

b. $\frac{w^{-2}x^4}{2w^{-5}}$

$$\frac{w^{-2}x^4}{2w^{-5}} = \frac{1}{2}\left(\frac{w^{-2}}{w^{-5}}\right)x^4$$ Group powers that have the same base.

$$= \frac{1}{2}(w^{-2-(-5)})x^4$$ Quotient of powers

$$= \frac{1}{2}w^3x^4$$ Simplify.

You can divide a polynomial by a monomial by separating the terms of the numerator.

EXAMPLE

2 **Simplify** $\frac{15x^3 - 3x^2 + 12x}{3x}$.

$$\frac{15x^3 - 3x^2 + 12x}{3x} = \frac{15x^3}{3x} - \frac{3x^2}{3x} + \frac{12x}{3x}$$ Divide each term by $3x$.

$$= 5x^2 - x + 4$$ Simplify.

Division can sometimes be performed using factoring.

EXAMPLE

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3 **Find** $(n^2 - 8n - 9) \div (n - 9)$.

$$(n^2 - 8n - 9) \div (n - 9) = \frac{n^2 - 8n - 9}{(n - 9)}$$ Write as a rational expression.

$$= \frac{(n - 9)(n + 1)}{(n - 9)}$$ Factor the numerator.

$$= \frac{\cancel{(n - 9)}(n + 1)}{\cancel{(n - 9)}}$$ Divide by the GCF.

$$= n + 1$$ Simplify.

When you cannot factor, you can use a long division process similar to the one you use in arithmetic.

EXAMPLE

4 **Find $(n^3 - 4n^2 - 9) \div (n - 3)$.**

In this case, there is no n term, so you must rename the dividend using 0 as the coefficient of the missing term.

$(n^3 - 4n^2 + 9) \div (n - 3) = (n^3 - 4n^2 + 0n + 9) \div (n - 3)$

Divide the first term of the dividend, n^3, by the first term of the divisor, n.

$$\begin{array}{rl}
 & n^2 - n - 3 \\
n - 3 \overline{)} & n^3 - 4n^2 + 0n + 12 \\
\underline{(-)n^3 - 3n^2} & \text{Multiply } n^2 \text{ and } n - 3. \\
-n^2 + 0n & \text{Subtract and bring down } 0n. \\
\underline{(-)-n^2 + 3n} & \text{Multiply } -n \text{ and } n - 3. \\
-3n + 12 & \text{Subtract and bring down } 12. \\
\underline{(-)-3n + 9} & \text{Multiply } -3 \text{ and } n - 3. \\
3 & \text{Subtract.}
\end{array}$$

Therefore, $(n^3 - 4n^2 + 9) \div (n - 3) = n^2 - n - 3 + \frac{3}{n - 3}$. Since the quotient has a nonzero remainder, $n - 3$ is not a factor of $n^3 - 4n^2 + 9$.