Multiplying Polynomials

The **Product of Powers** rule states that for any number a and all integers m and n, $a^m \cdot a^n = a^{m+n}$.

EXAMPLE

1 **Simplify each expression.**

a. $(4p^5)(p^4)$

$$\begin{aligned}(4p^5)(p^4) &= (4)(1)(p^5 \cdot p^4)\\ &= (4)(1)(p^{5+4})\\ &= 4p^9\end{aligned}$$

b. $(3yz^5)(-9y^2z^2)$

$$\begin{aligned}(3yz^5)(-9y^2z^2) &= (3)(-9)(y \cdot y^2)(z^5 \cdot z^2)\\ &= -27(y^{1+2})(z^{5+2})\\ &= -27y^3z^7\end{aligned}$$

The Distributive Property can be used to multiply a monomial by a polynomial.

EXAMPLE

2 **Simplify $3x^3(-4x^2 + x - 5)$.**

$$\begin{aligned}3x^3(-4x^2 + x - 5) &= 3x^3(-4x^2) + 3x^3(x) - 3x^3(5) && \text{Distributive Property}\\ &= -12x^5 + 3x^4 - 15x^3 && \text{Multiply.}\end{aligned}$$

To find the power of a power, multiply the exponents. This is called the **Power of a Power** rule.

EXAMPLE

3 **Simplify each expression.**

a. $(-3x^2y^4)^3$

$$\begin{aligned}(-3x^2y^4)^3 &= (-3)^3(x^2)^3(y^4)^3\\ &= -27x^6y^{12}\end{aligned}$$

b. $(xy)^3(-2x^4)^2$

$$\begin{aligned}(xy)^3(-2x^4)^2 &= x^3y^3(-2)^2(x^4)^2\\ &= x^3y^3(4)x^8\\ &= 4x^3 \cdot x^8 \cdot y^3\\ &= 4x^{11}y^3\end{aligned}$$

To multiply two binomials, find the sum of the products of

F the *First* terms,
O the *Outer* terms,
I the *Inner* terms, and
L the *Last* terms.

EXAMPLE

4 **Find $(2x - 3)(x + 1)$.**

$$\begin{aligned}(2x - 3)(x + 1) &= \underset{\text{F}}{(2x)(x)} + \underset{\text{O}}{(2x)(1)} + \underset{\text{I}}{(-3)(x)} + \underset{\text{L}}{(-3)(1)} && \text{FOIL method}\\ &= 2x^2 + 2x - 3x - 3 && \text{Multiply.}\\ &= 2x^2 - x - 3 && \text{Combine like terms.}\end{aligned}$$

The Distributive Property can be used to multiply any two polynomials.

EXAMPLE

5 **Find $(3x - 2)(2x^2 + 7x - 4)$.**

$$(3x - 2)(2x^2 + 7x - 4) = 3x(2x^2 + 7x - 4) - 2(2x^2 + 7x - 4) \quad \text{Distributive Property}$$
$$= 6x^3 + 21x^2 - 12x - 4x^2 - 14x + 8 \quad \text{Distributive Property}$$
$$= 6x^3 + 17x^2 - 26x + 8 \quad \text{Combine like terms.}$$

Three special products are $(a + b)^2 = a^2 + 2ab + b^2$,
$(a - b)^2 = a^2 - 2ab + b^2$, and
$(a + b)(a - b) = a^2 - b^2$.

EXAMPLE

6 **Find each product.**

a. $(2x - z)^2$

$$(a - b)^2 = a^2 - 2ab + b^2 \quad \text{Square of a difference}$$
$$(2x - z)^2 = (2x)^2 - 2(2x)(z) + (z)^2 \quad a = 2x \text{ and } b = z$$
$$= 4x^2 - 4xz + z^2 \quad \text{Simplify.}$$

b. $(3x + 7)(3x - 7)$

$$(a + b)(a - b) = a^2 - b^2 \quad \text{Product of sum and difference}$$
$$(3x + 7)(3x - 7) = (3x)^2 - (7)^2 \quad a = 3x \text{ and } b = 7$$
$$= 9x^2 - 49 \quad \text{Simplify.}$$